THEORETICAL AND EXPERIMENTAL INVESTIGATION OF THE FUNCTION OF THE WALL FLOW DEFLECTING RING. A MATHEMATICAL MODEL FOR THE CASE OF A LARGE NUMBER OF DEFLECTING RINGS

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Solution of the mathematical model has been developed of the distribution of liquid in a random packed bed in the presence of the wall flow deflecting rings. The solution was obtained in the form of a recurrent analytical formula. The formula permits, by repetitive calculation, profiles to be obtained of the densities of irrigation in a bed equipped with an arbitrary number of the wall flow deflecting rings. A comparison of experimental profiles of the density of irrigation with theoretical profiles showed considerable discrepancies between the two profiles. A new optimization of the parameters of the model, taking also the width of the wall flow deflecting ring as an adjustable parameter, showed a good agreement provided that the optimum width of the deflecting ring is about 1·5 times its real physical dimension. This finding can be explained by interaction of the function of the deflecting ring with the packing elements immediately adherring to the ring. This explanation may be partially supported by the visual observation showing the liquid to be deflected by the elements of the packing contacting the deflecting ring instead of the ring alone.

The drawback of the packed beds – the tendency to form the wall flow – can be overcome by using the wall flow deflecting rings[1]. These rings are placed in the column in various distances one from another and contact the column wall. This has been confirmed by several experimental studies[2–5] showing the mass transfer coefficient in the columns equipped with the wall flow deflecting rings to be independent of the column diameter and the depth of the packed section. The width of the wall flow deflecting rings to be independent of the column diameter and the depth of the packed section. The width of the wall flow deflecting rings (WFDR) is comparable with the size of an element of the packing while their spacing along the column height is usually large (200–400 mm). Both these facts indicate that the increase of the column costs, associated with the implementation of the WFDRs is negligible making their use potentially attractive.

On the other hand, the development of the methods for the calculation of the optimum spacing of the WFDRs in dependence on their width and the type of the mass transfer operation to be carried out in the column is not quite straightforward. A first step in this direction was made in paper[6], presenting a mathematical model of the distribution of liquid in a packed bed column in the presence of a single WFDR, together with the verification of its adequacy. There it has been also shown that for modelling of the effect of the WFDR one can utilize the differential equation of Cihla and Schmidt[7], together with the boundary conditions[8], expressing the imperfect capability of the wall to totally deflect the liquid and the tendency to wall flow formation by means to two coefficients[9].

The aim of the present work is to present a mathematical model of the distribution process in the column in the presence of a large number of WFDRs.

THEORY

The basic differential equation governing the distribution of liquid in the bed[7] for the case of axially symmetric flows takes the following dimensionless form:

$$\frac{\partial^2 f(r, Z)}{\partial r^2} + \frac{1}{r}\frac{\partial f(r, Z)}{\partial r} = \frac{\partial f(r, Z)}{\partial Z}. \qquad (1)$$

The effect of the wall containing the bed is expressed by the following boundary condition[8]:

$$-\frac{\partial f(r, Z)}{\partial r} = B[f(r, Z) - CW], \quad r = 1, \qquad (2)$$

where the coefficient B expresses the intensity of exchange of liquid between the flow trickling down the packing and the wall flow trickling down the surface of the wall. C characterizes the conditions when a hydrodynamic equilibrium between the wall flow and that on the packing has been reached. Such conditions exist theoretically near the bottom of the column of infinite depth of the packed section.

The general solution of Eq. (1), together with the boundary condition (2), takes the following form:

$$f(r, Z) = A_0 + \sum_n A_n \, J_0(q_n r) \exp(-q_n^2 Z), \qquad (3)$$

where q_n are roots of the following characteristic equation:

$$\left(\frac{2C}{q_n} - \frac{q_n}{B}\right) J_1(q_n) + J_0(q_n) = 0. \qquad (4)$$

The coefficient A_0 is determined by the following equation:

$$A_0 = C/(1 + C). \qquad (5)$$

The coefficients A_n are yet to be determined from the initial condition, *i.e.* the initial distribution of liquid, given in geneal by a function $\gamma(r)$, see ref.[10] as follows:

$$A_n = \frac{2((q_n^2/B) - 2C)^2}{[(q_n^2/B) - 2C)^2 + q_n^2 + 4C]\, J_0^2(q_n)} \int_0^1 \gamma(r)\, r\, J_0(q_n r)\, dr\,. \qquad (6)$$

Let us assume now that there is a large number of WFDRs in the column spaced evenly along the height of the packed section, the spacing being designated as Z_0, see Fig. 1. Similarly as in our previous paper[6], we shall assume that all wall flow, together with the liquid on the packing, that hits the WFDR is deflected and leaves WFDR on its internal radius. The initial condition for each section of the packed

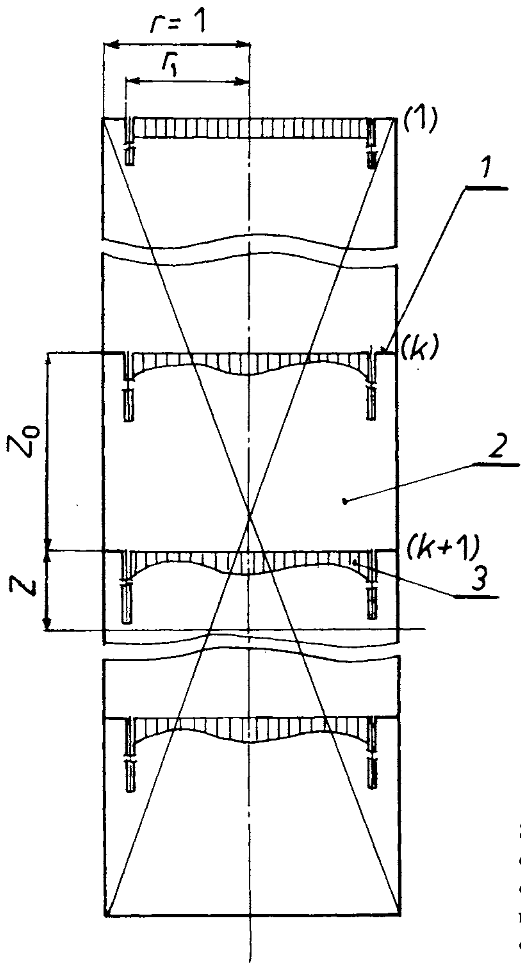

FIG. 1

Sketch of the column with the wall flow deflecting rings and resulting distribution of density of irrigation. *1* wall flow deflecting ring, *2* packing, *3* distribution of density of irrigation layer appears thus the solution (*3*) for the height $Z = Z_0$ for the immediately preceding section of the packed layer. In this way the initial condition for the section of the packed layer below the $(k + 1)$-th WFDR is given by the following expressions:

$$\gamma(r, Z) = f^{(k)}(r, Z)_{Z=Z_0} =$$
$$= A_0 + \sum_n A_n^{(k)} \mathrm{J}_0(q_n r) \exp(-q_n^2 Z_0) \quad \text{for} \quad Z = 0, \quad 0 \leqq r < r_1 \tag{7}$$

$$\gamma(r, Z) = \infty \quad \text{for} \quad Z = 0, \quad r = r_1 \tag{8}$$

$$\gamma(r, Z) = 0 \quad \text{for} \quad Z = 0, \quad r_1 < r \leqq 1. \tag{9}$$

In addition, based on the mass balance, we may write that:

$$2 \int_0^1 \gamma(r, Z)_{Z=0}\, r \, \mathrm{d}r = 1. \tag{10}$$

After substituting the initial conditions (*7–9*) into Eq. (*6*) one obtains an expression for the coefficients $A_n^{(k+1)}$ determining the solution (*3*) in the packed section below the $(k + 1)$-th WFDR. The integral in Eq. (*6*) is evaluated as follows:

$$\int_0^1 \gamma(r)\, r\, \mathrm{J}_0(q_n r)\, \mathrm{d}r = \lim_{\varepsilon \to 0} \left\{ \int_0^{r_1 - \varepsilon} [A_0 + \sum_n A_n^{(k)} \mathrm{J}_0(q_n r) \exp(-q_n^2 Z_0)]\, r\, \mathrm{J}_0(q_n r)\, \mathrm{d}r \right\} +$$
$$+ \lim_{\varepsilon \to 0} \int_{r_1}^{r_1 + \varepsilon} \gamma(r_1, 0)\, r\, \mathrm{J}_0(q_n r)\, \mathrm{d}r =$$
$$= A_0 (r_1 / q_n)\, J_1(q_n r_1) + \sum_{m \neq n} A_m^{(k)} \exp(-q_m^2 Z_0) \lim_{\varepsilon \to 0} \left[\int_0^{r_1 - \varepsilon} \mathrm{J}_0(q_m r)\, \mathrm{J}_0(q_n r)\, r\, \mathrm{d}r \right] +$$
$$+ A_n^{(k)} \exp(-q_n^2 Z_0) \lim_{\varepsilon \to 0} \left[\int_0^{r_1 - \varepsilon} \mathrm{J}_0^2(q_n r)\, r\, \mathrm{d}r \right] + \frac{J_0(q_n r_1)}{2} \lim_{\varepsilon \to 0} \left[\int_{r_1}^{r_1 + \varepsilon} 2\gamma(r, 0)\, r\, \mathrm{d}r \right]. \tag{11}$$

For the evaluation of the first two integrals in Eq. (*11*) one has to keep in mind the properties of the Bessel functions:

$$\int_a^b \mathrm{J}_0(q_m r)\, \mathrm{J}_0(q_n r)\, r\, \mathrm{d}r = \frac{1}{q_m^2 - q_n^2} \left| [q_n r\, \mathrm{J}_0(q_m r)\, \mathrm{J}_0'(q_n r) - q_m r\, \mathrm{J}_0(q_n r)\, \mathrm{J}_0'(q_m r)] \right|_a^b \tag{12}$$

$$\int_a^b \mathrm{J}_0^2(q_n r)\, r\, \mathrm{d}r = \left| \frac{r^2}{2} \{ \mathrm{J}_0^2(q_n r) + [\mathrm{J}_0'(q_n r)]^2 \} \right|_a^b \tag{13}$$

$$\mathrm{J}_0'(x) = -\mathrm{J}_1(x). \tag{14}$$

In addition, from the mass balance (*10*) and from the initial condition (*9*), we have that:

$$\lim_{\varepsilon \to 0} \left[\int_{r_1}^{r_1+\varepsilon} 2r\gamma(r, 0)\, \mathrm{d}r \right] = 1 - \lim_{\varepsilon \to 0} \left[\int_0^{r_1-\varepsilon} 2\gamma(r, 0)\, r\, \mathrm{d}r \right] =$$

$$= 1 - \lim_{\varepsilon \to 0} \left\{ \int_0^{r_1-\varepsilon} 2\left[A_0 + \sum_n A_n^{(k)} \mathrm{J}_0(q_n r) \exp\left(-q_n^2 Z_0\right)\right] r\, \mathrm{d}r \right\} =$$

$$= 1 - A_0 r_1^2 - 2r_1 \sum_n A_n^{(k)} \exp\left(-q_n^2 Z_0\right) \frac{\mathrm{J}_1(q_n r_1)}{q_n}. \qquad (15)$$

Upon expressing the integral (*11*) with the aid of the expressions (*12–15*) and upon substituting the result into Eq. (*6*) one obtains the coefficients $A_n^{(k+1)}$ determining the solution (*3*) in the packed section below the $(k + 1)$-th WFDR in the form:

$$A_n^{(k+1)} = \frac{2\left((q_n^2/B) - 2C\right)^2}{\left[\left((q_n^2/B) - 2C\right)^2 + q_n^2 + 4C\right] \mathrm{J}_0^2(q_n)} \left\{ \frac{C r_1}{(1 + C)\, q_n} \mathrm{J}_1(q_n r_1) + \right.$$

$$+ \sum_{m \neq n} r_1 A_m^{(k)} \exp\left(-q_m^2 Z_0\right) \frac{1}{q_m^2 - q_n^2} \left[q_m \mathrm{J}_0(q_n r_1)\, \mathrm{J}_1(q_m r_1) - q_n \mathrm{J}_0(q_m r_1)\, \mathrm{J}_1(q_n r_1)\right] +$$

$$+ A_n^{(k)} \exp\left(-q_n^2 Z_0\right) \frac{r_1^2}{2} \left[\mathrm{J}_0^2(q_n r_1) + \mathrm{J}_1(q_n r_1)\right] +$$

$$\left. + \frac{\mathrm{J}_0(q_n r_1)}{2} \left[1 - \frac{C r_1^2}{1 + C} - 2r_1 \sum_n A_n^{(k)} \exp\left(-q_n^2 Z_0\right) \frac{\mathrm{J}_1(q_n r_1)}{q_n}\right] \right\}. \qquad (16)$$

The obtained recurrent formula (*16*) permits one to calculate the distribution of liquid in the packed section below an arbitrary WFDR by repetitive evaluation of the coefficients, starting from the earlier obtained[6] solution below the first WFDR, *i.e.* the one located on the very top of the packed layer. Although this procedure is associated with a considerable computational effort, a program written in FORTRAN for a computer handled this problem quite easily.

EXPERIMENTAL

The experiments were carried out on a set-up described in the earlier paper[6]. The column was a tube 188·6 mm in internal diameter. The liquid distributor fed the liquid uniformly on the top of the packed section by means of 236 capillary tubes. At the bottom the liquid was collected by 4 concentric tubes. The radii of these tubes, delimiting anullar collecting surfaces, are given in Table I.

The packed layer was formed by Rashing rings 25 × 25 × 3 mm. Three configurations of the WFDRs were studied. The number of the WFDRs and their spacing for the three configurations are given in Table II. The width of the WFDRs in all cases studied was 20 mm. Also in all cases the first WFDR was placed on the top of the packed layer.

The experiments were carried out with water at two densities of irrigation, namely $1{\cdot}67 \, . \, 10^{-3}$ $m^3/(m^2s)$ and $3{\cdot}30 \, . \, 10^{-3}$ $m^3/(m^2s)$ with 6 repeated experiments for each configuration. The packed layer was repacked prior to each repeated run.

In order to avoid serious errors a careful analysis of random errors in experimental data has been carried out. Having in mind the different surface areas of the liquid collecting sections, the analysis of uniformity of the variance of the experiments has been carried out first using the Cochran criterion[12]. Next, a careful analysis has been carried out of the results for each configuration in each collecting section, while grouping the data into pairs with respect to the repacking and the density of irrigation (see p. 536 of ref.[13]). This approach has led to elimination of about 11% of experimental data which is acceptable considering the normal scatter of similar results.

The variance of reproducibility was determined (p. 635 of ref.[13]) from:

$$S_0^2 = \frac{1}{n - m} \sum_{i=1}^{m} (n_i - 1) S_i^2 \,, \tag{17}$$

where

$$S_i^2 = \frac{1}{n_i - 1} \sum_{k=1}^{n_i} (\bar{J}_{ik} - \bar{J}_i)^2$$

$$n = \sum_{i=1}^{m} n_i \quad \text{and} \quad \bar{J}_i = \frac{1}{n_i} \sum_{k=1}^{n_i} \bar{J}_{ik} \,.$$

For our experiment $S_2^0 = 0{\cdot}0534$ with 52 degrees of freedom.

RESULTS

The calculations were carried out on an EC computer. For each experiment were evaluated from the model the mean density of irrigation in each of the collecting section. For the outermost segment (IV) the density of irrigation was computed from the corresponding flow rate on the packing plus the wall flow divided by the area of the collecting segment. The coefficient of the radial spread of liquid was taken to be equal 0·00235 m, in accord with ref.[6], which appears optimal for the

TABLE I

Radii delimiting the liquid collecting sections at the bottom

Section designation	I	II	III	IV
Internal radius	0	40·2	55·3	88·9
External radius	40·2	55·3	88·9	94·3

Rashig rings. The coefficients of the boundary condition were taken $B = 7{\cdot}0$, $C = 1{\cdot}365$, in accord with ref.[9].

The results of experiments are shown in Table III together with the standard deviation. The residual variance was determined (ref.[13], p. 635) from:

$$S_A^2 = \frac{1}{m-1} \sum_{i=1}^{m} n_i(\bar{J}_i - \bar{J}_{i,c})^2 . \tag{18}$$

In the given case $S_A^2 = 0{\cdot}413$ with 11 degrees of freedom. The adequacy of the model was tested by the Fisher criterion (ref.[13], p. 636):

$$S_A^2/S_0^2 = 7{\cdot}73 > F_{1-\alpha/2} = 2{\cdot}0 \tag{19}$$

TABLE II

Experimental configurations

Configuration designation	Number of WFDR	Spacing of WFDR, mm
1	3	100
2	2	100
3	2	200

TABLE III

Results of experiments

Configuration	Segment	$\bar{J}_i$	$\bar{J}_{i,c}$	δ, %
1	I	1·38	1·12	22·6
1	II	1·26	1·15	9·5
1	III	0·87	0·86	0·8
1	IV	0·71	1·24	−42·9
2	I	1·27	1·09	16·4
2	II	1·28	1·14	12·6
2	III	0·83	0·87	−4·9
2	IV	0·99	1·28	−22·4
3	I	1·18	1·06	11·7
3	II	1·13	1·00	12·6
3	III	0·81	0·75	8·0
3	IV	1·41	2·13	−33·7

at the significance level $\alpha = 0{\cdot}10$. Consequently, the above model is inadequate.

The results of experiments and calculations show that maximum deviations are observed in the collecting section adherring to the wall. In all cases these deviations are negative with respect to the experiment. In other words, the flow near the wall is less than predicted by the model. The WFDRs in reality deflect the liquid more effectively toward the column axis and thus function as if their effective width was greater than the geometrical one.

Following this conclusion an attempt has been made to optimize the results while taking the width of the WFDR as an adjustable parameter. The results of these optimizations showed in all cases the optimum width of the WFDR to be about 50% greater than the real geometrical width. Naturally, the increase of the effective width of the WFDR depends apparently on the size of the ring as well as the size of the packing element. The factor of effective increase of the width of the WFDR, 1·5, thus cannot be taken as universal. The fact that the coefficient of effective increase of the width of the WFDR for all experiments carried out gave practically the same value, independently of the number of the WFDRs, seems to confirm the basis of the model proposed. It seems that the source of the descrepancies between the experimental results and the model calculations, based on the geometrical width of the WFDR, is the interaction of the deflecting function of the WFDR proper and that of the packing elements adherring to the internal periphery of the WFDR. Thus the doubt is cast only on the initial condition of the original model stipulating that the liquid deflected by the WFDR leaves it on its inner periphery. In contrast, this stipulation was found correct in the earlier paper[6] for the case of the single WFDR placed directly on top of the packed section with no elements of packing resting on the WFDR.

Both these findings lead to the conclusion that when the WFDR is located within the packed section the principal part of liquid does not leave it on its internal periphery but, instead, is deflected by the packing elements in the immediate vicinity of the WFDR. These elements though may deflect the liquid on different radii depending on the angle of their inclination. In other words, the presence of the WFDR disturbs the structure of bed with the resulting tendency to predominantly towards the axis oriented flow. Visual observation of the flow of liquid in a perspex glass column confirmed the above speculation. The liquid that hits the WFDR drains from its surface predominantly via the packing elements contacting the WFDR. This mechanism is readily enhanced by the good wettability of ceramic rings by the flowing liquid.

It may be concluded that the applied model may serve well as long as proper effective dimensions of the WFDR are used. A more theoretically founded calculation of the spread of liquid near the WFDR, however, mandates analysis of the microstructure of the packed bed in this region. This shall be subject of the following communication.

LIST OF SYMBOLS

A_0, A_n	coefficients in Eq. (*3*)
B, C	dimensionless parameters of the boundary condition (*2*)
D	coefficient of radial spread of liquid, m
F	Fischer criterion
$f = L/L_0$	dimensionless density of irrigation
$\bar{f}$	mean dimensionless density of irrigation
h	height of packed section measured from previous WFDR, m
h_0	spacing of WFDR, m
J_0, J_1	Bessel function of the first kind, zero and first order
k	sequence number of WFDR
L, L_0	local and mean density of irrigation, $m^3/(m^2s)$
m	number of configurations, summation index
n	number of experiments, summation index
q_n	roots of Eq. (*4*)
R	column radius, m
r'	radial coordinate, m
$r = r'/R$	dimensionless radial coordinate
r_1	dimensionless internal radius of WFDR
S^2	variance
W	dimensionless wall flow
$Z = Dh/R^2$	dimensionless axial coordinate
$Z_0 = Dh_0/R^2$	dimensionless axial spacing of WFDRs
δ	relative percentual deviation of mean density of irrigation for i-th collecting section
ε	infinitesimal quantity
γ	initial density of irrigation distribution function

Superscripts

(k)	k-th WFDR

Subscripts

A	residual variance
c	calculated value
i	i-th collecting section
k	k-th repeated experiment
0	variance

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